Journal of Engineering Physics and Thermophysics, Vol. 78, No. 2, 2005

INERTIAL SETTLING OF PARTICLES FROM A GAS-DISPERSED FLOW ON A CURVILINEAR SURFACE

K. N. Volkov and V. N. Emel'yanov UDC 532.529:536.24

Consideration is given to problems of settling of particles of a dispersed impurity from a subsonic jet flow on a curvilinear surface. Flow of the carrier medium is modeled based on the equations of vortex flow of a nonviscous incompressible liquid; they are written in the stream function–velocity vortex variables. The interaction of plane and round jets with obstacles of different shapes for the zero and nonzero initial vorticities of the flow is modeled numerically. The coefficient of sedimentation of the impurity is calculated as a function of the particle size and the obstacle shape.

Introduction. The problem on the influence of the particle size and the shape of the surface in flow on the intensity of inertial settling of an impurity from a subsonic jet flow is of interest for many practical applications, in particular, for the technology of gas-flame deposition of coatings.

In many cases of practical importance, the influence of viscosity on the flow field and the generation of turbulence are negligible and manifest themselves only in a thin layer adjacent to the obstacle surface [1–3]. The characteristics of the region of rotation of the jet are mainly determined by the balance of pressure forces resulting from the flow angularity and the inertial forces of the flowing liquid. The liquid in the region of turn of the flow can be taken ideal in a fairly wide class of problems, and the hereditary effects of viscosity can be allowed for by introduction of a vortex velocity profile at entry into the region of vortex flow. The accuracy of the results obtained based on the nonviscous-liquid model not infrequently proves to be comparable to or even higher than the accuracy of the results of solution of Navier–Stokes equations [4]. It becomes necessary to allow for the viscous properties of the carrier medium only in computation of the characteristics of resistance of a particle.

The theory of jets of an ideal liquid is an important and thoroughly developed division of hydromechanics in which consideration is given to flows bounded by moving and fixed solid walls and free surfaces the pressure along which is constant [5].

In incidence of round jets on an obstacle at an angle different from a right angle, a three-dimensional flow appears. The results of numerical calculations show that the flow field in the plane of symmetry is close to a two-dimensional flow [4]. When the slope of the obstacle is small, the isobars are close to circles and the streamlines are close to radial rays [6].

In this work, consideration is given to problems of inertial settling of dispersed-impurity particles from a subsonic jet flow on the surface of a curvilinear obstacle. For numerical modeling of subsonic jet flows with free boundaries we use the boundary-element method based on curvilinear grid structures [7, 8]. The shape of the free jet surface is found using the methods of solution of extremum problems [8, 9]. The motion of the dispersed phase is described within the framework of the Lagrangian approach. The sedimentation coefficient of the impurity is calculated as a function of the particle size and the obstacle shape.

Carrier Phase. Let us consider the stationary interaction of the jet of a weightless ideal incompressible liquid flowing out of a nozzle of size h with parallel walls and an infinitely large obstacle installed at distance H from the nozzle exit section at an angle α to the x axis (Fig. 1). It is assumed that the nozzle exit section is at a sufficiently large distance from the site of impingement of the jet upon the surface and the presence of the obstacle exerts no influence on the upstream development of flow. In the longitudinal direction, the computational domain is bounded by

---

D. F. Ustinov Baltic State Technical University "Voenmekh", 1 1ya Krasnoarmeiskaya Str., St. Petersburg, 190005, Russia; email: dsci@mail.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 78, No. 2, pp. 93–98, March–April, 2005. Original article submitted January 29, 2004.

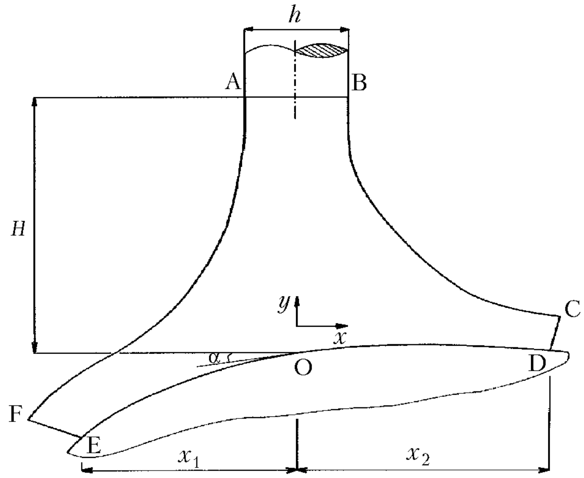

Fig. 1. Diagram of flow and the coordinate system.

the cross sections x_1 and x_2. The origin of the coordinate system represents the point of intersection of the median of the jet and the surface of the obstacle. The types of two-dimensional problems solved differ in geometry. The problem is considered to be axisymmetric for $\alpha = 0$ and plane for $\alpha \neq 0$. In impingement of the jet upon the obstacle at a right angle, calculation is carried out in the domain $x \geq 0$.

Basic Calculation Relations. Flow in the jet is described based on the nonviscous-flow equations written in the stream function–velocity vortex variables for a nonzero initial vorticity of the flow. Relations describing the flow have the form [8]

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - \frac{s}{x} \frac{\partial \psi}{\partial x} = - x^s \omega \,, \tag{1}$$

$$\frac{\omega}{x^s} = c_1 (\psi) \,. \tag{2}$$

Here we have $s = 0$ and $s = 1$ for the plane and axisymmetric problems respectively. The vorticity is found from the relation

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \,.$$

The velocity components are related to the stream function by the formulas

$$u = \frac{1}{x^s} \frac{\partial \psi}{\partial y} \,, \quad v = - \frac{1}{x^s} \frac{\partial \psi}{\partial x} \,.$$

Pressure is computed from the Bernoulli equation

$$\frac{u^2 + v^2}{2} + \frac{p}{\rho} = c_2 (\psi) \,. \tag{3}$$

The presence of the integral of (2) enables us to find ω from the condition of constancy of the vorticity along the streamline for $s = 0$ or the condition of constancy of the ratio of the vorticity to the distance from the axis of symmetry along the streamline for $s = 1$ [4, 10] and not to solve the equation of vortex transfer.

We select the nozzle width (jet width at infinity) as the characteristic scale for the variables with dimensions of length and the value of the velocity and the total pressure on the median of the jet for the velocity and the pressure.

Boundary Conditions. The solution of Eq. (1) is sought in the domain bounded by the nozzle exit section AB (inlet boundary), the surfaces AF and BC (free boundaries) separating a moving liquid from the medium at rest, the free-outflow boundaries FE and CD (outlet boundaries) located at a sufficiently large distance from the critical point O, and the solid wall ED (Fig. 1).

The calculations were carried out for several initial velocity profiles corresponding to the linear dependence of the velocity on coordinates and to the main portion of the plane and round jets [10]. The prescribed dependence of the velocity vortex on the stream function at entry completely characterizes the vorticity distribution in the entire region of flow [4].

Since the flow is steady-state, the boundaries AF and BC represent the stream surfaces along which ψ = const. The value of the stream function at the free boundary is computed from the velocity distribution on the nozzle exit section. A distinctive feature of setting up the boundary conditions at the boundaries AF and BC is that the condition of constancy of the pressure is specified instead of the free-surface shape. Taking account of the assumption on the weightlessness of the liquid and the Bernoulli equation (3), we replace the condition of constancy of the pressure by the condition of constancy of the modulus of the velocity vector on the free surface [8]. We took the line on which the velocity-vector modulus was equal to $c_3 V_m$ ($c_3 \sim 0.1$) as the free surface of the jet in the calculations. The conditions of equilibrium outflow $\partial\psi/\partial n = 0$ or the condition of constancy of the velocity-vector modulus were set up at the outlet boundaries FE and CD.

Dispersed Phase. We use the Lagrangian approach to model the motion of dispersed-impurity particles. Equations describing the translational motion of a spherically shaped probe particle have the form

$$\frac{d\mathbf{r}_p}{dt} = \mathbf{v}_p \,, \tag{4}$$

$$\frac{d\mathbf{v}_p}{dt} = \frac{3C_d\rho}{8\rho_p r_p} \left|\mathbf{v} - \mathbf{v}_p\right| (\mathbf{v} - \mathbf{v}_p) \,. \tag{5}$$

The coefficient of drag is represented as follows:

$$C_d = \frac{24}{\mathrm{Re}_p} (1 + 0.179\,\mathrm{Re}_p^{0.5} + 0.013\,\mathrm{Re}_p) \,.$$

The Reynolds number in the relative motion of the particle and the liquid is found from the formula

$$\mathrm{Re}_p = \frac{2r_p\rho \left|\mathbf{v} - \mathbf{v}_p\right|}{\mu} \,.$$

Equations (4) and (5) are integrated along the trajectory of an individual particle and require that only the initial conditions — coordinates of the particle and its velocity at the instant of time $t = 0$ — be specified. Consideration is given to the longitudinal injection of particles onto the nozzle exit section with a rate equal to the velocity of outflow of the jet.

Particle motion is determined by the value of the Stokes number, equal to the ratio of the characteristic time of the flow to the characteristic time of relaxation of the particle velocity, $\mathrm{Stk} = 2r_p^2\rho_p V_{m,a}/(9h\mu)$.

Computational Procedure. Equation (1) is solved in a curvilinear domain matched to the boundaries in the physical space of the coordinate system; the lines of the level of functions that are the solution of a system of elliptical-type equations are taken as its coordinate lines [8, 11].

The boundary-element method [7, 8] is used for numerical integration of the Poisson equation (1). As a result, the Poisson equation is transformed to an integral equation determining the boundary values of the stream function.

The stream function is computed inside the domain using the solution obtained at the boundary. The dimensionality of the problem solved decreases by unity, which leads to a substantial reduction in the number of discrete elements as compared to the finite-difference method requiring that the entire domain, including the boundary, be discretized. The methods of solution of extremum problems [8, 9] are used to seek the free jet surface.

The trajectories of motion of a particle can be calculated in both physical and computational spaces. In physical space, difficulties with recording of the instant at which the particle is incident on the wall or escapes from the outlet cross section of the computational domain arise. The main difficulty of integration of the equations of motion of the particle in a curvilinear coordinate system is associated with interpolation of the grid values of the parameters of the carrier flow. However, knowing the position of the particle in the physical plane, we can determine in which cell of the curvilinear grid it is located and can take the corresponding velocity value for interpolation. Organization of computations in the (ξ, η) space is more simple, since it becomes unnecessary to reveal the cell of the grid structure that is next in the direction of particle motion. The accuracy of integration is reduced in view of the fact that the metric parameters of the grid template are not, strictly speaking, analytical functions.

Integration of Eqs. (4) and (5) is simultaneous in two spaces — physical and computational ones. Since the equations for coordinates are not related to the equations for velocity, we integrate the equations for velocity components in the physical space (x, y) and the equations for coordinates in the computational plane (ξ, η).

The equations for the coordinates of the particle in the curvilinear coordinate system have the form

$$\frac{d\xi}{dt} = \frac{1}{x^s J}\left(u\frac{\partial y}{\partial \eta} - v\frac{\partial x}{\partial \eta}\right), \tag{6}$$

$$\frac{d\eta}{dt} = \frac{1}{x^s J}\left(v\frac{\partial y}{\partial \xi} - u\frac{\partial y}{\partial \xi}\right). \tag{7}$$

The Jacobian of transformation of the coordinates is found from the relation

$$J = \frac{\partial x}{\partial \xi}\frac{\partial y}{\partial \eta} - \frac{\partial x}{\partial \eta}\frac{\partial y}{\partial \xi}.$$

We use the implicit Euler method to integrate Eqs. (6) and (7). Formulas ensuring the continuity of the interpolated quantity when the particle passes from one cell of the grid into another are constructed based on the weighted-area method:

$$x_i = \sum_{i=1}^{4} S_i x_i \Big/ \sum_{i=1}^{4} S_i .$$

By S we mean the area of one of the rectangles into which the grid cell is subdivided, in the computational space, by lines that are parallel to the coordinate axes and pass through the point at which the particle is located.

Calculation Results. In accordance with the model of nonviscous vortex flow, the character of flow inside the zone of integration of the jet with the obstacle is totally determined by the prescribed velocity and vorticity distributions in the initial cross section of the jet and by the conditions of parallelism of the flow at the outlet boundaries of the computational domain [4, 6]. The results of numerical calculations for the flow field of the carrier medium are in quite satisfactory agreement with the data of physical experiment, with the theory describing vortex flow of an ideal liquid near the critical point, and with the available numerical solutions [4, 8]. Slight disagreements with the experimental data are due to the presence of mixing in the zone of rotation of the flow [8, 10]. The results of the corresponding calculations and a detailed description of the computational algorithm have been given in [8].

Let us consider the settling of alumina particles on flat, concave, and convex surfaces of spherical shape (for R = 1.2 m and α = 0). The jet on the nozzle exit section has a uniform velocity profile and a zero vorticity. The results of calculations of the trajectories of particles are shown in Fig. 2 for different Stokes numbers. The trajectories

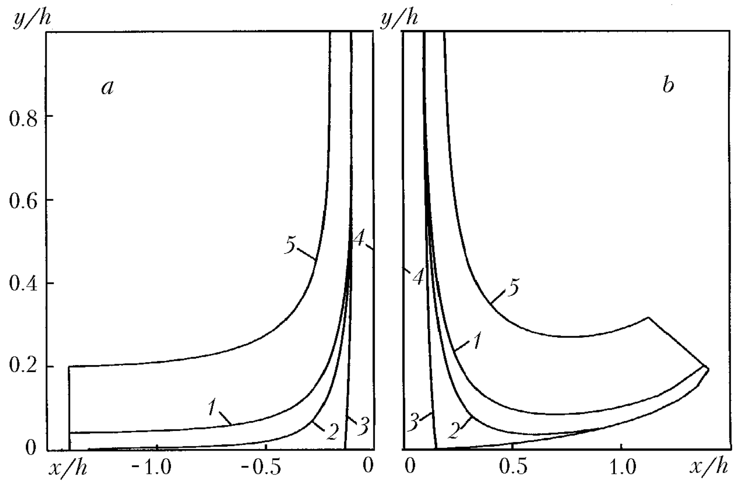

Fig. 2. Trajectories of particles in incidence of the jet on flat (fragment a) and concave (fragment b) obstacles for Stk = 0.1 (1), 1.0 (2), and 2.5 (3); curve 4 corresponds to the critical streamline, and curve 5 corresponds to the free surface.

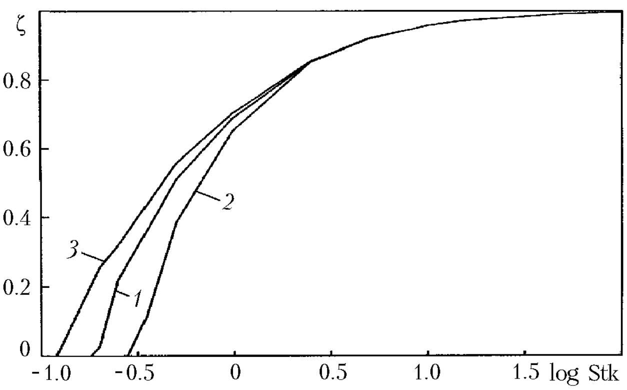

Fig. 3. Sedimentation coefficient of the impurity vs. Stokes number in incidence of the jet on flat (1), convex (2), and concave (3) surfaces.

of hydrodynamically small particles (Stk = 0.1) follow, in practice, the shape of the streamlines of the carrier medium (curve 1). When Stk = 1.0 particles collide with the obstacle surface at a fairly small angle (curve 2). The trajectories of hydrodynamically large particles (Stk = 2.5) are not deflected, in practice, by the obstacle and collide with it at a nearly right angle (curve 3).

In the inertial regime of settling where all the incident particles are absorbed by the surface, the intensity of settling of the particles is characterized by the sedimentation coefficient, by which we mean the ratio of the density of the flux of incident particles at the critical point to the density of the flux of the particles of a given fraction in an unperturbed flow:

$$\zeta = \left(\frac{x_{p0}}{x_{w0}} \right)^{s+1} .$$

Calculation requires that the trajectories of a particle passing through a certain point x_{p0} in the unperturbed flow and meeting with the surface at the point $x_{p,w}$ be known.

The dependence of ζ of the impurity on Stk is shown in Fig. 3 for obstacles of flat, concave, and convex shapes (for the zero vorticity of the flow on the nozzle exit section). In each case we have a critical value of Stk below which the settling of the impurity is absent. The value of the sedimentation coefficient of particles for the convex obstacle turns out to be lower (curve 2) than that in the case of settling of the impurity on the flat surface (curve 1) and higher for the concave obstacle (curve 3).

The results of the calculations performed for the nonzero vorticity of the flow on the nozzle exit section (we prescribed the linear velocity profile at entry into the computational domain, which, by virtue of the relation (2), corresponds to the condition ω = const in the entire flow field) demonstrate the change in the intensity of settling of the impurity. Particles introduced into the jet flow with velocity $V_{m,a} = (V_{left} + V_r)/2$ acquire different accelerations in different regions of the jet, due to which we have a change in the intensity of settling of the dispersed phase as compared to the case $\omega = 0$. When $V_{left} < V_r$, the values of the sedimentation coefficient decrease on the left of the critical point and increase on the right.

It is noteworthy that the problem on settling of particles in the vicinity of the critical point of the obstacle in the case $\omega_a \neq 0$ (and consequently in the entire flow field) requires additional investigations based on a more detailed grid or it requires that analytical methods be used. For the nonzero vorticity of the flow at entry into the computational domain, the critical streamline of the jet bends in the immediate vicinity of the obstacle surface [4, 6], which can trigger the intensification of settling of the impurity.

Conclusions. We have developed numerical modeling aids enabling us to predict the intensity of inertial settling of particles of a dispersed impurity from the jet flow of a nonviscous incompressible liquid, which is incident on an arbitrarily shaped obstacle for the zero and nonzero vorticities of the flow on the nozzle exit section. Numerical implementation of the model, based on the method of boundary integral equations, allows application of software that has already been developed to solution of three-dimensional problems.

NOTATION

c, constant; C_d, drag coefficient; h, width of the outlet cross section of the nozzle, m; H, distance from the nozzle exit section to the obstacle, m; J, Jacobian of transformation of the coordinates, n, external normal; p, pressure, Pa; r, radius, m; $\mathbf{r}$, radius vector, m; R, radius of the obstacle, m; Re, Reynolds number; s, index of the flow geometry; S, area, m^2; Stk, Stokes number; t, time, sec; u and v, velocity components, m/sec; $\mathbf{v}$, velocity vector, m/sec; V, modulus of the velocity vector, m/sec; x, y, spatial coordinates, m; α, angle between the tangent to the obstacle surface at the critical point and the x axis; ζ, sedimentation coefficient; μ, dynamic viscosity, kg/(m·sec); ξ, η, curvilinear coordinates, m; ρ, density, kg/m^3; ψ, stream function; ω, vorticity. Subscripts: a refers to the nozzle exit section; d refers to the characteristics of drag of a particle; i, dummy index; left refers to the left-hand boundary of the jet; m refers to the median of the jet; p, particle; r refers to the right-hand boundary of the jet; w, obstacle surface; 0, initial instant of time.

REFERENCES

1. W. Wolfshtein, Some solutions of the plane turbulent impinging jet, *J. Basic Eng.*, **92**, No. 4, 915–922 (1970).
2. W. W. Bower and D. R. Kotansky, A Navier–Stokes analysis of the two-dimensional ground effects problem, *AIAA Paper*, No. 76-621 (1976).
3. W. W. Bower, Computations of three-dimensional impinging jets based on the Reynolds equations, *AIAA Paper*, No. 82-1024 (1982).
4. A. Rubel, Computations of jet impingement on a flat surface, *AIAA J.*, **18**, No. 2, 168–175 (1980).
5. M. I. Gurevich, *Theory of Ideal-Liquid Jets* [in Russian], Fizmatlit, Moscow (1961).
6. A. Rubel, Computations of the oblique impingement of round jets upon a plane wall, *AIAA J.*, **19**, No. 7, 863–871 (1981).
7. C. Brebbia, J. Telles, and L. Wrobel, *Boundary Element Techniques* [Russian translation], Mir, Moscow (1987).
8. V. A. Anisimov, K. N. Volkov, and V. N. Emel'yanov, Subsonic jet flows with free boundaries, *Mat. Modelir.*, **11**, No. 12, 16–32 (1999).
9. F. P. Vasil'ev, *Numerical Methods for Solving Extremum Problems* [in Russian], Nauka, Moscow (1983).
10. H. Schlichting, *Boundary-Layer Theory* [Russian translation], Nauka, Moscow (1974).
11. S. K. Godunov, A. V. Zabrodin, M. Ya. Ivanov, A. N. Kraiko, and G. P. Prokopov, *Numerical Solution of Multidimensional Problems of Gas Dynamics* [in Russian], Nauka, Moscow (1979).